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Chapter 6 Test Review

Describe in what situations it is appropriate to use u-substitution, integration by parts, and tabular.

U sub
* See a func. & a version of its derivative
- The func. is on INSIDE of composite

By Parts
* Product of funcs.
- U sub does NOT work
- use LIPET to pick u

Tabular
* By Parts
- u is a polynomia

Regular Integration 1. $\int\left(3x^4 - \frac{2}{x^3} + \sec^2 x + \frac{1}{e^x}\right)dx = \boxed{\frac{3}{5}x^5 + \frac{1}{x^2} + \tan x - \frac{1}{e^x} + C}$

$\int(3x^4 - 2x^{-3} + \sec^2 x + e^{-x})dx$

U sub 2. $\int \sqrt{\csc 7x}\,\csc 7x \cot 7x\,dx = -\frac{1}{7}\int u^{\frac{1}{2}}du = -\frac{1}{7}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \boxed{-\frac{2}{21}(\csc 7x)^{\frac{3}{2}} + C}$

$u = \csc 7x$

$du = -7\csc 7x \cot 7x\,dx \to dx = \frac{-1}{7\csc 7x \cot 7x}du$

Parts or Tab. 3. $\int x^3 e^{-2x}dx$

$\boxed{-\frac{1}{2}x^3 e^{-2x} - \frac{3}{4}x^2 e^{-2x} - \frac{3}{4}x e^{-2x} - \frac{3}{8}e^{-2x} + C}$

u	dv
x^3	e^{-2x}
$3x^2$	$-\frac{1}{2}e^{-2x}$
$6x$	$\frac{1}{4}e^{-2x}$
6	$-\frac{1}{8}e^{-2x}$
0	$\frac{1}{16}e^{-2x}$

Parts $uv - \int v\,du$ 4. $\int_{e^2}^{e^5} x\ln x\,dx = \frac{1}{2}x^2\ln x - \int\frac{1}{2}x^2\left(\frac{1}{x}dx\right) = \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2\Big]_{e^2}^{e^5}$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$= \left[\frac{1}{2}(e^5)^2\ln e^5 - \frac{1}{4}(e^5)^2\right] - \left[\frac{1}{2}(e^2)^2\ln e^2 - \frac{1}{4}(e^2)^2\right]$

$= \frac{5}{2}e^{10} - \frac{1}{4}e^{10} - e^4 + \frac{1}{4}e^4 = \boxed{\frac{9}{4}e^{10} - \frac{3}{4}e^4}$

U sub 5. $\int\frac{\sqrt{\ln r}}{r}dr = \int u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}} + C = \boxed{\frac{2}{3}(\ln r)^{\frac{3}{2}} + C}$

$u = \ln r$

$du = \frac{1}{r}dr \to dr = r\,du$

U sub 6. $\int\frac{1}{\theta^2}\sin\frac{1}{\theta}d\theta = -\int\sin u\,du = --\cos u + C = \boxed{\cos\left(\frac{1}{\theta}\right) + C}$

$u = \frac{1}{\theta}$ OR θ^{-1}

$du = -\theta^{-2}d\theta \to d\theta = -\theta^2 dr$

U sub 7. $\int_1^2\frac{x+5}{x^2+10x+6}dx = \frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(x^2+10x+6)\Big]_1^2 = \frac{1}{2}\ln(2^2+10\cdot2+6) - \frac{1}{2}\ln(1^2+10$

$u = x^2+10x+6$

$du = 2x+10\,dx \to dx = \frac{1}{2(x+5)}du$

$= \boxed{\frac{1}{2}(\ln 30 - \ln 17)}$

$= \boxed{\frac{1}{2}\ln\frac{30}{17}}$

Regular Integration 8. $\int\frac{dt}{t\sqrt{t}} = \int t^{-\frac{3}{2}}dt = -2t^{-\frac{1}{2}} + C = \boxed{\frac{-2}{\sqrt{t}} + C}$

$t\cdot t^{\frac{1}{2}}$

Parts $uv - \int v\,du$ 9. $\int x^4\ln x\,dx = \frac{1}{5}x^5\ln x - \int\frac{1}{5}x^5\left(\frac{1}{x}dx\right) = \frac{1}{5}x^5\ln x - \frac{1}{5}\int x^4dx = \boxed{\frac{1}{5}x^5\ln x - \frac{1}{25}x^5 + C}$

$u = \ln x \quad dv = x^4dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{5}x^5$

U sub 10. $\int 5e^x e^{-6e^x}dx = -\frac{5}{6}\int e^u du = -\frac{5}{6}e^u + C = \boxed{-\frac{5}{6}e^{-6e^x} + C}$

$u = -6e^x$

$du = -6e^x dx \to dx = \frac{1}{-6e^x}du$

U sub. 11. $\int_0^1\frac{dy}{y-2} = \int\frac{1}{u}du = \ln|u| = \ln|y-2|\Big]_0^1 = \ln|1-2| - \ln|0-2|$

$u = y-2$

$du = dy$

$= \ln 1 - \ln 2$

$= \boxed{-\ln 2}$

NAME_______________________

Chapter 6 Test Review

12. If $y(1) = 0.25$, solve $\frac{dy}{dx} = -2xy^2$. Then write an equation for the tangent line at the given point.

$\frac{1}{y^2}dy = -2x\,dx$

$\int y^{-2}dy = \int -2x\,dx \rightarrow -y^{-1} = -x^2 + C$

If find C now:

$-\frac{1}{.25} = -(1)^2 + C$

$-4 = -1 + C \rightarrow C = -3 \rightarrow -\frac{1}{y} = -x^2 - 3$

$\frac{1}{y} = x^2 + 3$

$\boxed{y = \frac{1}{x^2+3}}$

Tangent Line

Slope

$\frac{dy}{dx} = -2(1)\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$

$y = -\frac{1}{8}(x-1) + \frac{1}{4}$

* 13. Find the equation for position of a particle whose acceleration (in m/sec²) is modeled by $a(t) = 2 + 6t$. At t=0, velocity is 4 m/sec. Find the velocity as a function of time t. How far does the particle move during the first second of its trip, from t=0 to t=1.

At t=1, position is -2 m.

$v(t) = 2t + 3t^2 + C_1$

$4 = 2(0) + 3(0)^2 + C_1$

$4 = C_1$

$\rightarrow v(t) = 3t^2 + 2t + 4$

$s(t) = t^3 + t^2 + 4t + C$

$-2 = 1^3 + 1^2 + 4(1) + C$

$-2 = 6 + C \rightarrow C = -8$

$\rightarrow s(t) = t^3 + t^2 + 4t - 8$

$s(0) = -8$

$s(1) = -2$

∴ Particle moved 6 m.

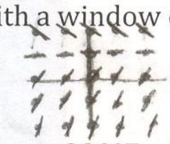
14. Find the area under the curve of $\sqrt{9x+1}$ from $x = -\frac{1}{9}$ to $x = \frac{1}{3}$.

$\text{Area} = \int_{-\frac{1}{9}}^{\frac{1}{3}} (9x+1)^{\frac{1}{2}}dx = \underbrace{\frac{1}{9}\cdot\frac{2}{3}}_{\frac{2}{27}}(9x+1)^{\frac{3}{2}} \rightarrow \frac{2}{27}(9x+1)^{\frac{3}{2}}\Big]_{-\frac{1}{9}}^{\frac{1}{3}} = \left[\frac{2}{27}\left(9\left(\tfrac{1}{3}\right)+1\right)^{\frac{3}{2}}\right] - \left[\frac{2}{27}\left(9\left(-\tfrac{1}{9}\right)+1\right)^{\frac{3}{2}}\right]$

$= \frac{2}{27}(8) - \frac{2}{27}(0) = \boxed{\frac{16}{27}}$

15. Construct a slope field for $\frac{dy}{dx} = 1 - y$ on a graph with a window of [-2,2] by [-2,2].

y	dy/dx
-2	3
-1	2
0	1
1	0
2	-1



16. A deep-dish apple pie, whose internal temperature was 220°F when removed from the oven, was set out on a 40°F breezy porch to cool. Fifteen minutes later, the pie's internal temperature was 180°F. How long did it take the pie to cool from there to 70°F?

$T - T_s = (T_0 - T_s)e^{-kt}$

$180 - 40 = (220 - 40)e^{-k(15)}$

$140 = 180e^{-15k}$

$\frac{7}{9} = e^{-15k}$

$\ln\frac{7}{9} = \ln e^{-15k} \rightarrow k = \frac{\ln\frac{7}{9}}{-15}$

$k \approx .01675$

$70 - 40 = (220 - 40)e^{-.01675t}$

$30 = 180e^{-.01675t}$

$\frac{1}{6} = e^{-.01675t}$

$\ln\frac{1}{6} = \ln e^{-.01675t} \rightarrow t = \frac{\ln\frac{1}{6}}{-.01675}$

$t \approx 106.94$

∴ It took $\boxed{\approx 1\text{ hr } \& \text{ } 31\text{ m}}$ to cool from 180° to 70°.

17. A pan of warm water (46°C) was put into a refrigerator. Ten minutes later, the water's temperature was 39°C; 10 minutes after that, it was 33°C. Use Newton's Law of Cooling to estimate how cold the refrigerator was.

$T_0 = 46°$ when $t = 10$, $T = 39°$; when $t = 20$ $T = 33°$

$39 - T_s = (46 - T_s)e^{-k(10)}$

$\frac{39 - T_s}{46 - T_s} = e^{-10k}$

$33 - T_s = (46 - T_s)e^{-k(20)}$

$\frac{33 - T_s}{46 - T_s} = e^{-20k}$

B/C of expo rules: $(e^{-10k})^2 = e^{-20k}$ → Using subst: $\left(\frac{39 - T_s}{46 - T_s}\right)^2 = \frac{33 - T_s}{46 - T_s}$

$\frac{(39 - T_s)^2}{(46 - T_s)^2} = \frac{33 - T_s}{46 - T_s}$

$(39 - T_s)^2(46 - T_s) = (33 - T_s)(46 - T_s)$

$1521 - 78T_s + T_s^2 = 1518 - 79T_s + T_s^2$

$\boxed{T_s = -3°}$

18. After applying for a credit card, you read in the fine print that it is compounded continuously at a rate of 6%. If you spend \$500 on a new IPad and put it on your credit card, how much will you owe if you don't pay off the IPad after 6 months (1/2 year)?

$A = Pe^{rt}$

$A = 500e^{.06(\frac{1}{2})}$

$\approx \$515.23$

Amount after 6 months if comp. cont. @ 6% will be $\boxed{\approx \$515.23.}$

19. Suppose an ice cube out of a freezer that was 28°F and put it outside on a 97°F day. If the rate at which the temperature of the ice changing every minute is given by $\frac{2e^x}{45}$, approximate what the temperature of the ice cube is 7 minutes later.

$\frac{dT}{dx} = \frac{2e^x}{45}$

$T(0) = 28$

$T(7) = ?$

$T(7) = T(0) + \int_0^7 \frac{2e^x}{45}dx$ ← use calcu

$\approx 28 + 48.69480$

$\boxed{\approx 76.695°}$