Name ______________________________ Period ______ Date ______________

AP Calculus ~ Worksheet 6.1 – 6.3

#1 – 2: Use integration by parts

1. $\int_0^{\frac{\pi}{3}} x\sin x\,dx$ $= -x\cos x + \int \cos x\,dx = -x\cos x + \sin x\Big]_0^{\frac{\pi}{3}}$

$u = x \quad \int dv = \int \sin x\,dx$

$du = dx \quad v = -\cos x$

$= \left[-\frac{\pi}{3}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] - \left[\cancel{0\cos 0} + \cancel{\sin 0}\right]$

$= -\frac{\pi}{3}\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}$

$= \boxed{-\frac{\pi}{6} + \frac{\sqrt{3}}{2}}$

2. $\int_1^e \ln x\,dx$ $= x\ln x - \int x\left(\frac{1}{x}dx\right) = x\ln x - x\Big]_1^e$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$= \left[e\,\cancel{\ln e}^{1} - e\right] - \left[\cancel{1\ln 1}^{0} - 1\right]$

$= e - e - 0 + 1$

$= \boxed{1}$

#3: Use substitution (Trig. Identity)

3. $\int \cos^2 x\,dx$ $= \int \frac{1+\cos 2x}{2}dx = \int \frac{1}{2}dx + \frac{1}{2}\int \cos 2x\,dx = \frac{1}{2}x + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + C$

$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$

#4 – 6: Use u-substitution

4. $\int x\cos\left(x^2\right)dx$ $= \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C = \boxed{\frac{1}{2}\sin x^2 + C}$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

5. $\int (3x-8)^{49}\,dx$ $= \frac{1}{3}\int u^{49}du = \frac{1}{3}\left(\frac{u^{50}}{50}\right) + C = \boxed{\frac{1}{150}(3x-8)^{50} + C}$

$u = 3x-8$

$du = 3dx$

$\frac{1}{3}du = dx$

6. $\int \frac{dx}{x\ln x}$ $= \int \frac{1}{u}du = \ln|u| + C = \boxed{\ln|\ln x| + C}$

$u = \ln x$

$du = \frac{1}{x}dx$

$dx = x\,du$

Solve the initial value problem.

7. $\frac{dy}{dt} = 6e^t$ where $y(\ln 3) = 0$

$t = \ln 3 \quad y = 0$

$\int dy = \int (6e^t)\,dt$

$y = 6e^t + C$

$0 = 6e^{\cancel{\ln 3}} + C$

$0 = 6 \cdot 3 + C$

$C = -18$

$\boxed{y = 6e^t - 18}$

8. $\int a(t) = \int 3\sin t$ velocity at t = 0 is 1 and the position at t = 0 is -1.
Find the equation for the body's velocity and position at any time, t.

$v(t) = -3\cos t + C$

$1 = -3\cos 0 + C$

$1 = -3 + C$

$4 = C$

$\int v(t) = \int -3\cos t + 4$

$s(t) = -3\sin t + 4t + C$

$-1 = -\cancel{3\sin 0}^{0} + \cancel{4 \cdot 0}^{0} + C$

$-1 = C$

v(t) = $-3\cos t + 4$

s(t) = $-3\sin t + 4t - 1$

9.) $\int x^2 \cos\left(\frac{x}{2}\right)dx$

D: $f(x) = x^2$	I: $g(x) = \cos\left(\frac{x}{2}\right)$
x^2	$\cos\left(\frac{x}{2}\right)$
$2x$	$2\sin\left(\frac{x}{2}\right)$
2	$-4\cos\left(\frac{x}{2}\right)$
0	$-8\sin\left(\frac{x}{2}\right)$

$\boxed{2x^2\sin\left(\frac{x}{2}\right) + 8x\cos\left(\frac{x}{2}\right) - 16\sin\left(\frac{x}{2}\right) + C}$

10.) $\int t^2 \ln t\, dt = \frac{t^3}{3}\ln t - \int \frac{t^3}{3}\left(\frac{1}{t}dt\right)$

$u = \ln t \quad \int dv = \int t^2 dt$

$du = \frac{1}{t}dt \quad v = \frac{t^3}{3}$

$= \frac{t^3}{3}\ln t - \frac{1}{3}\int t^2 dt$

$= \frac{1}{3}t^3 \ln t - \frac{1}{3}\left(\frac{1}{3}t^3\right) + C$

$= \boxed{\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3 + C}$

11.) $\int_0^5 x^2\sqrt{x^3 - 10}\,dx$ (Hint: use u-substitution)

$0 \quad 3$

$u = x^3 - 10$

$du = 3x^2 dx$

$\frac{1}{3}du = x^2 dx$

$u(3) = 27 - 10 = 17$

$u(5) = 125 - 10 = 115$

$= \frac{1}{3}\int_{17}^{115} u^{\frac{1}{2}}du = \frac{1}{3}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big]_{17}^{115} = \frac{2}{9}u^{\frac{3}{2}}\Big]_{17}^{115}$

$= \frac{2}{9}(115)^{\frac{3}{2}} - \frac{2}{9}(17)^{\frac{3}{2}}$

$\boxed{\approx 258.477}$

12.) $\int_0^{\frac{\pi}{2}} x^3 \cos 2x\, dx$ (Hint: use tabular integration)

D: $f(x) = x^3$	I: $g(x) = \cos 2x$
x^3 (+)	$\cos 2x$
$3x^2$ (−)	$\frac{1}{2}\sin 2x$
$6x$ (+)	$-\frac{1}{4}\cos 2x$
6 (−)	$-\frac{1}{8}\sin 2x$
0	$\frac{1}{16}\cos 2x$

$\frac{1}{2}x^3\sin 2x + \frac{3}{4}x^2\cos 2x - \frac{3}{4}x\sin 2x - \frac{3}{8}\cos 2x\Big]_0^{\frac{\pi}{2}}$

$\left[\frac{1}{2}\left(\frac{\pi}{2}\right)^3\cancel{\sin 2\left(\frac{\pi}{2}\right)}^{0} + \frac{3}{4}\left(\frac{\pi}{2}\right)^2\cos 2\left(\frac{\pi}{2}\right) - \frac{3}{4}\left(\frac{\pi}{2}\right)\cancel{\sin 2\left(\frac{\pi}{2}\right)} - \frac{3}{8}\cos 2\left(\frac{\pi}{2}\right)\right]$

$-\left[\cancel{\frac{1}{2}(0)^3\sin 2(0)}^{0} + \cancel{\frac{3}{4}(0)^2\cos 2(0)}^{0} - \cancel{\frac{3}{4}(0)\sin 2(0)}^{0} - \frac{3}{8}\cos 2(0)\right]$

$-\frac{3\pi^2}{16} + \frac{3}{8} - \frac{3}{8} = \boxed{\frac{-3\pi^2}{16}}$

13.) $\int_{-3}^{2} e^{-2x}\sin 2x\,dx$

$u = e^{-2x} \quad dv = \sin 2x\,dx$

$du = -2e^{-2x}dx \quad v = -\frac{1}{2}\cos 2x$

(Hint: use integration by parts)

$-\frac{1}{2}e^{-2x}\cos 2x - \int \frac{+1}{2}\cos 2x\,(\cancel{2}e^{-2x}dx)$

$= -\frac{1}{2}e^{-2x}\cos 2x - \int e^{-2x}\cos 2x\,dx$

$u = e^{-2x} \quad dv = \cos 2x\,dx$

$du = -2e^{-2x}dx \quad v = \frac{1}{2}\sin 2x$

$= -\frac{1}{2}e^{-2x}\cos 2x - \frac{1}{2}e^{-2x}\sin 2x + \int \frac{1}{\cancel{2}}\sin 2x\,(-\cancel{2}e^{-2x}dx)$

$\int e^{-2x}\sin 2x\,dx = -\frac{1}{2}e^{-2x}\cos 2x - \frac{1}{2}e^{-2x}\sin 2x - \int e^{-2x}\sin 2x\,dx$

$+\int e^{-2x}\sin 2x\,dy \qquad + \int e^{-2x}\sin 2x\,dx$

$2\int e^{-2x}\sin 2x\,dx = -\frac{1}{2}e^{-2x}(\cos 2x + \sin 2x)$

$\int_{-3}^{2} e^{-2x}\sin 2x\,dx = -\frac{1}{4}e^{-2x}(\cos 2x + \sin 2x)\Big]_{-3}^{2}$

$= \left[-\frac{1}{4}e^{-2(2)}(\cos 2\cdot 2 + \sin 2\cdot 2)\right] - \left[-\frac{1}{4}e^{-2(-3)}(\cos 2(-3) + \sin 2(-3))\right]$

$= \boxed{-\frac{e^{-4}}{4}(\cos 4 + \sin 4) + \frac{e^{6}}{4}(\cos(-6) + \sin(-6))}$

≈ 125.02