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Chapter 6 Quiz 6.1-6.2 Review

Find the general solutions to the differential equations. Then, if given an initial condition, find that particular solution.

1. $\int \frac{dy}{dx} = \int 8x^5 - 2x^3 + 5x - 1; (1,5)$

$y = \frac{4}{3}x^6 - \frac{1}{2}x^4 + \frac{5}{2}x^2 - x + C$

$5 = \frac{4}{3} - \frac{1}{2} + \frac{5}{2} - 1 + C$

$5 = \frac{8}{6} - \frac{3}{6} + \frac{15}{6} - \frac{6}{6} + C$

$\frac{30}{6} = \frac{14}{6} + C \rightarrow C = \frac{16}{6} = \frac{8}{3}$

$\boxed{y = \frac{4}{3}x^6 - \frac{1}{2}x^4 + \frac{5}{2}x^2 - x + \frac{8}{3}}$

2. $\int \frac{dy}{dx} = \int -20 \sec 5x \tan 5x$

$y = \frac{-20 \tan 5x}{5} + C$

$\boxed{y = -4\tan 5x + C}$

3. $\int \frac{dy}{dx} = \int \frac{9}{x} + \frac{9}{x^2} + \frac{9}{x^3} = \frac{9}{x} + 9x^{-2} + 9x^{-3}$

$\boxed{y = 9\ln x - \frac{9}{x} - \frac{9}{2x^2} + C}$

4. $\int \frac{d^2y}{dx^2} = \int 4x + 1$

$\int \frac{dy}{dx} = \int 2x^2 + x + C_1$

$\boxed{y = \frac{2}{3}x^3 + \frac{1}{2}x^2 + C_1 x + C_2}$

5. $\int \frac{d^2y}{dx^2} = \int \cos x \,; y\left(\frac{\pi}{6}\right) = 0 \;\&\; y'(\pi) = 0$

$\frac{dy}{dx} = \sin x + C_1 \rightarrow \int \frac{dy}{dx} = \int \sin x$

$0 = \sin \pi + C_1$

$0 = 0 + C_1$

$C_1 = 0$

$y = -\cos x + C_2$

$0 = -\cos\left(\frac{\pi}{6}\right) + C_2$

$0 = -\frac{\sqrt{3}}{2} + C_2$

$C_2 = \frac{\sqrt{3}}{2}$

$\boxed{y = -\cos x + \frac{\sqrt{3}}{2}}$

6. $\int \frac{dy}{dx} = \int x\sqrt{x} + \sqrt[3]{x}; (0, -4)$

$x^{\frac{3}{2}} + x^{\frac{1}{3}}$

$y = \frac{2}{5}x^{\frac{5}{2}} + \frac{3}{4}x^{\frac{4}{3}} + C$

$-4 = 0 + C$

$C = -4$

$\boxed{y = \frac{2}{5}x^{\frac{5}{2}} + \frac{3}{4}x^{\frac{4}{3}} - 4}$

$\frac{2}{5}x^2\sqrt{x} + \frac{3}{4}x\sqrt[3]{x} - 4$

7. $\int \frac{dy}{dx} = \int \frac{-1}{1+x^2} - \frac{1}{\sqrt{1-x^2}}$

$\boxed{y = \cot^{-1} x + \cos^{-1} x + C}$

8. $\int \frac{dy}{dx} = \int e^{9x}$

$\boxed{y = \frac{1}{9}e^{9x} + C}$

9. $\frac{dy}{dx} = xy$

$\frac{1}{y}\,dy = x\,dx$

$\ln|y| = \frac{1}{2}x^2 + C$

e e

$|y| = e^{\frac{1}{2}x^2} \cdot e^C$ let $e^C = k$

$|y| = ke^{\frac{1}{2}x^2}$

$\rightarrow \boxed{y = \pm ke^{\frac{1}{2}x^2}}$

10. $\frac{dy}{dx} = -\frac{2x^5}{9y^2}; y = 2 \text{ when } x = -3$

$\int 9y^2\,dy = \int -2x^5\,dx$

$\frac{3y^3}{3} = -\frac{1}{3}x^6 + C$

$y^3 = -\frac{1}{9}x^6 + C_1$

$y = \sqrt[3]{-\frac{1}{9}x^6 + C_1}$

$2 = \sqrt[3]{-\frac{1}{9}(-3)^6 + C_1}$

$8 = -81 + C_1$

$89 = C_1$

$\boxed{y = \sqrt[3]{-\frac{1}{9}x^6 + 89}}$

If you want to find C right now:

$3(2)^3 = -\frac{1}{3}(-3)^6 + C$

$24 = -243 + C$

$267 = C$

$\frac{3y^3}{3} = -\frac{1}{3}x^6 + 267$

$y^3 = -\frac{1}{9}x^6 + 89$

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Chapter 6 Quiz 6.1-6.2 Review

Integrate.

11. $\int dx = \boxed{x + C}$

12. $\int 6\sqrt{5x+7}\,dx = 6\int\sqrt{5x+7}\,dx = \frac{6}{5}\int u^{\frac{1}{2}}du$

u sub: $u = 5x+7$

$du = 5dx \rightarrow dx = \frac{1}{5}du$

$= \frac{6}{5}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= \boxed{\frac{4}{5}(5x+7)^{\frac{3}{2}} + C}$

"Backwards Chain" since inside is linear: $6\int(5x+7)^{\frac{1}{2}}dx$

$= \frac{6(5x+7)^{\frac{3}{2}}}{\frac{2}{3}\cdot 5} + C$

13. $\int(5x^4 + 2x - 1)e^{(x^5+x^2-x-6)}dx$

u sub: $u = x^5 + x^2 - x - 6$

$du = (5x^4 + 2x - 1)dx \rightarrow dx = \frac{1}{5x^4+2x-1}dx$

$= \int e^u du$

$= e^u + C$

$= \boxed{e^{x^5+x^2-x-6} + C}$

14. $\int \frac{\ln x}{5x}dx = \frac{1}{5}\int\frac{\ln x}{x}dx = \frac{1}{5}\int u\,du$

u sub: $u = \ln x$

$du = \frac{1}{x}dx \rightarrow dx = x\,du$

$= \frac{1}{5}\cdot\frac{1}{2}u^2 + C$

$= \boxed{\frac{1}{10}\ln^2 x + C}$

15. $\int -\cos^7 x \sin x\,dx = \int u^7 du$

u sub: $u = \cos x$

$du = -\sin x\,dx \rightarrow dx = \frac{1}{-\sin x}du$

$= \frac{1}{8}u^8 + C$

$= \boxed{\frac{1}{8}\cos^8 x + C}$

16. $\int \cot x\,dx = \int\frac{\cos x}{\sin x}dx = \int\frac{1}{u}du$

$u = \sin x$

$du = \cos x\,dx \rightarrow dx = \frac{1}{\cos x}du$

$= \ln|u| + C$

$= \boxed{\ln|\sin x| + C}$

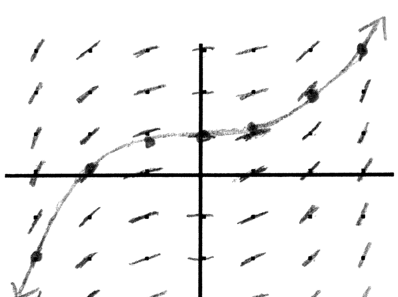
17. $\int(x-4)(2x+7)\,dx$

$= \int(2x^2 - x - 28)\,dx$

$= \boxed{\frac{2}{3}x^3 - \frac{1}{2}x^2 - 28x + C}$

Given the differential equations, draw their slope fields. Then, knowing that the graph of **#18 only** goes through (3,4), find the equation and graph it on your slope field.

18. $\frac{dy}{dx} = \frac{1}{3}x^2$



x	$\frac{dy}{dx} = \frac{1}{3}x^2$
-3	3
-2	$\frac{4}{3}$
-1	$\frac{1}{3}$
0	0
1	$\frac{1}{3}$
2	$\frac{4}{3}$
3	3

$\int\frac{dy}{dx} = \int\frac{1}{3}x^2$

$y = \frac{1}{9}x^3 + C$

$4 = \frac{1}{9}(3)^3 + C$

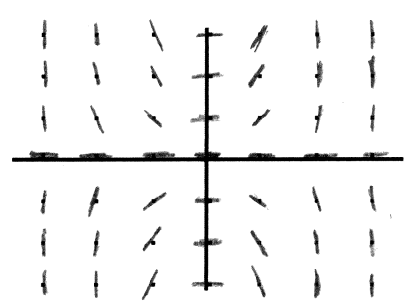
$4 = 3 + C$

$1 = C$

$y = \frac{1}{9}x^3 + 1$

-3	-2
-2	$\frac{1}{9}$
-1	$\frac{8}{9}$
0	1
1	$1\frac{1}{9}$
2	$1\frac{8}{9}$
3	3

19. $\frac{dy}{dx} = x^2 y$



x	y	$\frac{dy}{dx} = x^2y$
-3	-3	-27
-3	-2	-18
-3	-1	-9
-3	0	0
-3	1	9
-3	2	18
-3	3	27