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Chapter 5 Test Review

1. Write any word or phrase that is synonymous with "integral."
   Antiderivative, area under curve, net area, displacement

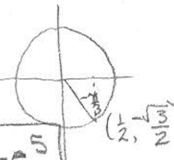
2. Write all the methods to ***approximate*** an integral. If there are any formulas, write those with them. LRAM, MRAM, RRAM, Trapezoids, Simpson's
   $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$; $S = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + \dots + 4y_{n-1} + \dots$

3. Write the Fundamental Theorem of Calculus.
   Part I: $F(x) = \int_a^x f(t)\,dt$; $\frac{dF}{dx} = \int_a^x f(t)\,dt = f(x)$ | Part II: $\int_a^b f(x)\,dx = F(a) - F(b)$

4. Compare and contrast definite integrals (see #5) and indefinite integrals (see #6)
   Comparison: Both use antiderivatives   Contrast: Definite find numerical answer (area under curve over interval). Indefinite find antiderivative equation. Answer must have +

5. Evaluate the integrals. Try without a calculator. Give ***EXACT*** answers.
   a. $\int_2^5 x\,dx = \frac{1}{2}x^2\Big]_2^5 = \left[\frac{1}{2}(5)^2\right] - \left[\frac{1}{2}(2)^2\right] = \frac{25}{2} - 2 = \frac{25}{2} - \frac{4}{2} = \boxed{\frac{21}{2}}$
   b. $\int_{-\pi/3}^{\pi} -\sin x\,dx = \cos x\Big]_{-\frac{\pi}{3}}^{\pi} = [\cos \pi] - [\cos(-\frac{\pi}{3})] = -1 - \frac{1}{2} = \boxed{-\frac{3}{2}}$

   sin
   -cos   cos
   -sin

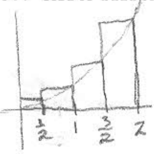
   

   c. $\int_1^6 e^{-x}\,dx = -e^{-x}\Big]_1^6 = [-e^{-6}] - [-e^{-1}] = \boxed{\frac{-1}{e^6} - \frac{1}{e}}$ OR $\frac{-1}{e^6} - \frac{e^5}{e^6} = \boxed{\frac{-1-e^5}{e^6}}$
   d. $\int_1^7 (2x^4 - 6)\,dx = \frac{2}{5}x^5 - 6x\Big]_1^7 = \left[\frac{2}{5}(7)^5 - 6(7)\right] - \left[\frac{2}{5}(1)^7 - 6(1)\right] = 6722\frac{4}{5} - 42 - \frac{2}{5} - 6$
   $= \boxed{6674\frac{2}{5}}$
   e. $\int_1^e \frac{1}{6x}\,dx = \frac{1}{6}\ln x\Big]_1^e = \left[\frac{1}{6}\cancel{\ln e}\right] - \left[\frac{1}{6}\cancel{\ln 1}^{\,0}\right] = \boxed{\frac{1}{6}}$

6. Integrate.
   a. $\int dx = \boxed{x + C}$
   b. $\int \sec^2 x\,dx = \boxed{\tan x + C}$
   c. $\int \frac{4}{x^7}\,dx = \int 4x^{-7}\,dx = -\frac{2}{3}x^{-6} = \boxed{\frac{-2}{3x^6} + C}$
   d. $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \boxed{\cos^{-1} x + C}$

For #7-9, consider the integral $\int_0^2 5x^4\,dx$. This function has too large of values to graph accurately.

7. Draw and find (without the program) the RRAM approximation using 4 partitions.



	W	x	Area
R_1	$\frac{1}{2}$	$f(\frac{1}{2}) = \frac{5}{16}$	$\frac{5}{32}$
R_2	$\frac{1}{2}$	$f(1) = 5$	$\frac{5}{2}$
R_3	$\frac{1}{2}$	$f(\frac{3}{2}) = \frac{405}{16}$	$\frac{405}{32}$
R_4	$\frac{1}{2}$	$f(2) = 80$	40

$\frac{5}{32} + \frac{5}{2} + \frac{405}{32} + 40 = \boxed{\frac{885}{16}}$
$= 55.3125$

8. Draw and find (without the program) the trapezoids approximation using 4 partitions.

$T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$
$= \frac{\frac{1}{2}}{2}\left[0 + 2(\frac{5}{16}) + 2(5) + 2(\frac{405}{16}) + 80\right]$
$= \frac{1}{4}\left(\frac{565}{4}\right)$
$= \boxed{\frac{565}{16}}$
$= 35.3125$

9. Find the Simpson's Rule approximations using 4 partitions.

$S = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + \dots + 4y_{n-1} + y_n)$
$= \frac{\frac{1}{2}}{3}\left[0 + 4(\frac{5}{16}) + 2(5) + 4(\frac{405}{16}) + 80\right)$
$= \frac{1}{6}\left(\frac{385}{2}\right)$
$= \boxed{\frac{385}{12}}$
$= 32.08\overline{3}$

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Chapter 5 Test Review

10. Find derivative using FTC Part 1.

a. $\int_x^7 5t \ln t \, dt = -\int_7^x 5t \ln t \, dt = \boxed{-5x \ln x}$

b. $\int_x^{x^2} \cos w \, dw = \int_x^0 \cos w \, dw + \int_0^{x^2} \cos w \, dw = -\int_0^x \cos w \, dw + \int_0^{x^2} \cos w \, dw = \boxed{-\cos x + 2x \cos(x^2)}$

c. $\int_0^{\ln x} p \, dp = \boxed{\frac{\ln x}{x}}$

11. Find the average value of the functions on the given intervals

$\frac{1}{b-a}\int_a^b f(x)\,dx$

a. $y = x^5$ on $[-5,7]$ $\quad \text{Avg} = \frac{1}{7--5}\int_{-5}^{7} x^5 \, dx = \frac{1}{12}\left(\frac{1}{6}x^6\Big]_{-5}^{7}\right) = \frac{1}{12}\left(\left[\frac{1}{6}(7)^6\right] - \left[\frac{1}{6}(-5)^6\right]\right)$
$= \frac{1}{12}(17004) = \boxed{1417}$

b. $y = \cos x$ on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ $\quad \text{Avg} = \frac{1}{\frac{3\pi}{2} - \frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx = \frac{1}{\pi}\left(\sin x\Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right) = \frac{1}{\pi}\left(\left[\sin\frac{3\pi}{2}\right] - \left[\sin\frac{\pi}{2}\right]\right) = \frac{1}{\pi}(-1-1)$
$= \boxed{\frac{-2}{\pi}}$

c. $y = \ln x$ on $[e, 2e]$ $\quad \text{Avg} = \frac{1}{2e-e}\underbrace{\int_e^{2e} \ln x \, dx}_{\text{calculator}} \approx \frac{1}{e}(3.76834) \boxed{\approx 1.386}$

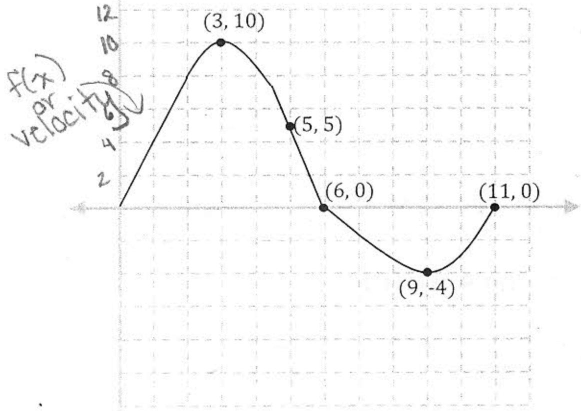
12. Let F(x) be an antiderivative of f(x). Use the initial condition to find F(7). If necessary, use your calculator.

a. $f(x) = 3x^2$; F(2)=6
$F(7) = F(2) + \int_2^7 3x^2$
$= 6 + x^3\Big]_2^7$
$= 6 + [7^3] - [2^3]$
$= 6 + 343 - 8 = \boxed{341}$
OR
$F(x) = \int 3x^2 = x^3 + C$
$6 = 2^3 + C$
$6 = 8 + C$
$-2 = C$
$F(x) = x^3 - 2$
$F(7) = 7^3 - 2$
$= 343 - 2$
$= \boxed{341}$

b. $f(x) = 9e^x$; F(0)=5
$F(7) = F(0) + \int_0^7 9e^x dx$
$= 5 + 9e^x\Big]_0^7$
$= 5 + [9e^7] - [9e^0]$
$= 5 + 9e^7 - 9$
$= \boxed{9e^7 - 4}$
OR
$F(x) = \int 9e^x = 9e^x + C$
$5 = 9e^0 + C$
$5 = 9 + C$
$C = -4$
$F(x) = 9e^x - 4$
$F(7) = \boxed{9e^7 - 4}$

c. $f(x) = \frac{\ln x}{1+x^2}$; F(1)=2
$F(7) = F(1) + \int_1^7 \frac{\ln x}{1+x^2} dx$
$\approx 2 + .49731$
$\boxed{\approx 2.497}$

13. The differentiable functions f(x) is shown below. The position of a particle is given by $s = \int_0^t f(x) \, dx$. Use the equation and graph to answer the following questions.



a. Find s(1). $= \int_0^1 f(x)\,dx \approx \frac{bh}{2} = \frac{1(4)}{2} = \boxed{2}$

b. Find the velocity at t=9. $\boxed{-4}$

c. Approximate the acceleration between t=5 and t=6. Accel. is der. $\boxed{-5}$

d. Is the acceleration positive or negative at t=10? $\boxed{\text{Positive}}$

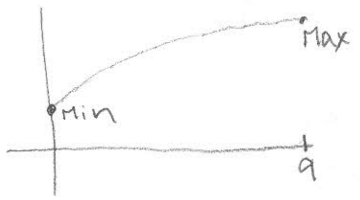
e. What is the particles approximate position at t=3? $s(3) = \int_0^3 f(x)\,dx \approx \frac{bh}{2} = \frac{3(10)}{2} = \boxed{15}$

f. When does s have the largest value? $\boxed{t=6}$

g. Approximately when is the acceleration zero? $\boxed{t=3, t=9}$

14. Suppose a bug is walking along a telephone wire with a velocity (in cm/s) modeled by $v(t) = 6\sqrt{x} + 3$. The bug walks for a total of 9 seconds and stops. Show that the bug must be between 27 cm and 189 cm from its original starting spot.

Min: $v(0) = 6\sqrt{0} + 3 = 3$
Max: $v(9) = 6\sqrt{9} + 3 = 21$



$\text{Min}\cdot(b-a) \le \int_a^b v(t)\,dt \le \text{Max}(b-a)$

$3(9-0) \le \int_0^9 v(t)\,dt \le 21(9-0)$

$\boxed{27 \text{ cm} \le \text{Actutual Position} \le 189 \text{ cm}}$