Limits & Continuity

- Evaluate limits algebraically, numerically, and graphically

- Understanding continuity in terms of limits

- Understanding asymptotes in terms of limits involving infinity

Limits are used to describe the behavior of a function's outputs "in the neighborhood" of a number x=*a,* where the function may or may not be defined. In order for the limit to exist, the function's outputs must "approach" the **same** number as you approach from the left and right of x=*a*. You can explore limits numerically (by looking at output values), graphically, and algebraically.

Consider the function $f(x)=\dfrac{x^3-8}{x-2}$. Let's investigate its behavior near $x=2$. We need to approach 2 from the right and from the left. Compute the following:

$f(2.1)$	$f(1.9)$
$f(2.01)$	$f(1.99)$
$f(2.001)$	$f(1.999)$
$f(2.0001)$	$f(1.9999)$

We can also gather evidence graphically by observing the behavior in the neighborhood of $x=2$.

The limit can be computed algebraically by factoring the numerator, and reducing the expression.

For each of the following, where $\lim_{x \to a} f(x)$

a) Evaluate $f(a)$ if it is defined
b) Evaluate the limit (numerically, graphically & algebraically, if possible) if it exists. Consider the one-sided limits.
c) Sketch the graph near x=a
d) Is $f(x)$ continuous at x=a?

A function f is **continuous** at x=a if all three of the following are true:

1. $\lim_{x \to a} f(x) = L$ (the limit exists)
2. $f(a) = M$ (the function is defined)
3. $L = M$

1. $\lim_{x \to 3}(2x-1)$

2. $\lim_{x \to 1} \frac{x^2-1}{x-1}$

3. $\lim_{x \to 2} \frac{x^3+x^2-11x+10}{x-2}$

4. $\lim_{x \to 0} \frac{|x|}{x}$

5. $\lim_{x \to 1} F(x)$, where $F(x) = \begin{cases} 2x+3, & x<1 \\ 4, & x=1 \\ x^2+1, & x>1 \end{cases}$

6. $\lim_{x \to 0}(1+x)^{\frac{1}{x}}$

7. $\lim_{x \to 1} g(x)$, where $g(x) = \begin{cases} \cos x, & x \le 1 \\ x^2+1, & x>1 \end{cases}$

8. $\lim_{x \to -1}([x]+2)$

9. $\lim_{x \to 4} \frac{1}{x-4}$

10. $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

11. $\lim_{x \to 0} \frac{\sin x}{x}$

12. $\lim_{x \to 0} \frac{(3+x)^2 - 9}{x}$

Find the limit (if it exists).

13. $\lim_{x \to \infty} \frac{1}{x}$

14. $\lim_{x \to \infty} \frac{-2x+4}{3x^2 - x + 1}$

15. $\lim_{x \to \infty} \frac{-2x^3+4}{3x^2 - x + 1}$

16. $\lim_{x \to \infty} \frac{-2x^2+4}{3x^2 - x + 1}$

For each of the following, sketch the graph of a function satisfying the stated requirements. Each problem is a separate graph.

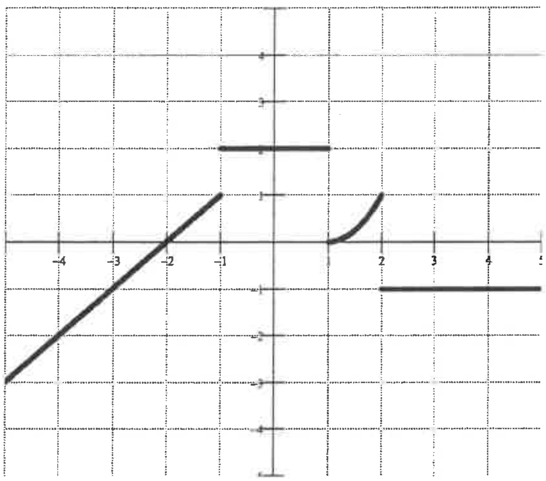
1. $\lim_{x \to 1^+} f(x) = 2$ $\lim_{x \to 1^-} f(x) = -1$ $f(1)$ *is undefined*

2. $\lim_{x \to -2^-} g(x) = 0$ $\lim_{x \to -2^+} g(x) = 0$ $g(-2) = 1$

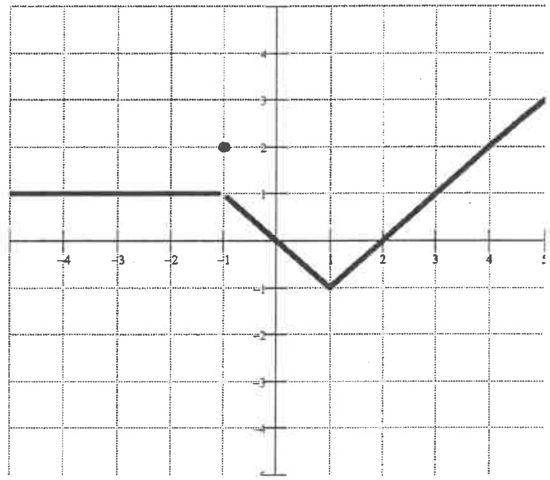
3. $\lim_{x \to 2^-} h(x) = -2$ $\lim_{x \to 2^+} h(x) = 2$ $h(2) = 0$

4. $\lim_{x \to 0^-} j(x) = -1$ $\lim_{x \to 0^+} j(x) = -2$ $j(0) = -1$

The graphs of the functions *f* and *g* are given below. Use the graphs to answer the following questions.



Graph of *f*



Graph of *g*

1. $\lim_{x\to -1^-} f(x)$

2. $\lim_{x\to -1^+} f(x)$

3. $\lim_{x\to -1} f(x)$

4. $\lim_{x\to -1^-} g(x)$

5. $\lim_{x\to -1^+} g(x)$

6. $\lim_{x\to -1} g(x)$

7. $\lim_{x\to 1} f(x)$

8. $\lim_{x\to 1} g(x)$

9. $\lim_{x\to -1} (f(x)g(x))$

10. $\lim_{x\to 0} (2f(x)+3g(x))$

11. $\lim_{x\to 0} \frac{f(x)}{g(x)}$

12. $\lim_{x\to 0} \frac{g(x)}{f(x)}$

13. $\lim_{x\to -2} g(f(x))$

14. $\lim_{x\to -1} f(g(x))$

15. $\lim_{x\to 0} (f(x)-10)$

16. $\lim_{x\to 0^-} f(x+2)$

17. $\lim_{x\to 0} (f(x-2)+4)$